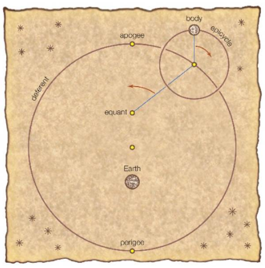
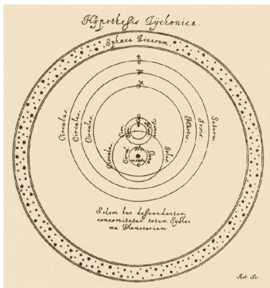
Thinking about the universe has been an old trade of humanity. Our perception and approach to the world have been changing as we evolve through ages and so has our understanding of the universe. It would be impossible to order how thoughts evolved from subject to subject precisely but, it seems that the first objects of the universe that the humanity has focused on have been the positioning of earth, the moon, the sun and a few other planets and stars with respect to each other. Where can you place all these celestial objects? Let us recall some answers to this question across centuries. Around 2BC, the people living around the ancient kingdom of Urartu in the eastern Caucasians thought of the universe as in figure 1a. This is a metal ornament with the earth at the bottom surrounded by a sphere for water and a sphere for the atmosphere with Mercury, Mars, Jupiter, Saturn and the Moon lined up between the Earth and the Sun. By the 2nd century AD, one of the conventional pictures of the universe was the Ptolemaic system with the Earth at the center of the Universe and the rest of the planets and stars going around in different circles, as in figure 1b.

(a) A bronze model of the Universe from ancient times. Source: History Museum of Armenia ticket



(b) Ptolemaic System for planetary motion. Source:Encyclopedia Britanica



(c) Tychonian System for planetary motion, picture source: Wikipedia

Figure 1: Some examples of modelling the universe

After the 16th century, with the works of Copernicus the Earth was no longer thought to be at the center of the Universe, new systems of spheres for the planets to move around were developed by Brahe and others. The ornament in figure 1a and the drawings in figures 1b and 1c are all models. They try to explain the observed motion of planets in simple terms as much as possible. In these examples the models are depicted in figurative terms. While these figurative descriptions were evolving as one language, another language being developed was the language of mathematical formulations, like the Kepler's laws of Planetary motion published between 1609 to 1619

1. The orbit of a planet is the ellipse. The Sun is positioned at one of the two foci of this ellipse.

2. The planets travel along this ellipse such that they scan equal areas at equal times.

3. The square of the orbital period T of a planet is related to cube of the semi-major axis a of the ellipse.

Formulations such as these, started from the level of trigonometry that most of us meet towards the end of elementary school to differential equations that only some of us encounter at the level of university. They provided the basis for Newton's laws of gravity published on 1687. When Newton's formulation of classical mechanics was established it became clear that second of Kepler's laws of planetary motion implied the conservation of angular momentum along the orbit. Angular momentum is a quantity associated with rotating objects. It is conserved in systems that do not change their behavior if the whole system is rotated along an axis. The property of not being affected by a rotation is called *angular symmetry.* There are many types of symmetries that are associated with some change in the description of a system that does not change the physical behavior of the system. The second law of Kepler's is an example to how symmetries are closely related with physical laws. In practice, symmetries help us to understand and formulate physical laws.

As much as humanity thought about the motion of the planets in a planetary system while gazing up at the night sky, it has also been concerned whether the universe is finite, and if it has always been in the same shape, in terms of distribution of planets. An important consequence of Copernicus' principal, the idea that the Earth is not at the center of the Universe, eventually led to the idea that there is no privileged observation point in the universe and no privileged direction. This is the idea that the universe is homogeneous and isotropic, and it is the idea that is at the heart of modern cosmology.

The night sky is full of many bright stars that seem to be infinite in number. Since the Greek monk Cosmas Indicopleustes of Alexandria put it into words, it has worried people that an infinite number of starts would mean a very bright and burning hot sky while the darkness of the night sky proved the situation to be quite the opposite. If the Universe was infinitely old, nothing in it ever changed (static), contained infinitely many stars over an infinite volume then there would be a star at any random line of sight. In short, in such a universe the night sky cannot be dark the way ours is. This is now referred to as the Olber's paradox. In contrast, it is very natural to have a dark night sky in an infinite universe whose volume and content evolves with time; giving each observer a finite sized region with a finite number of stars to be observed. This idea of a dynamic Universe, according to records initially mentioned in the essay Eureka by Edgar Allen Poe in 1848 and a paper by Lord Kelvin in 1901, has led us to our current understanding of the Universe. Since Hubble's observations of cepheid stars we know that our Universe is not static, it expands in time.

Our understanding of the Universe today is that it underwent different rates of expansion through out its history. Similar to the way jeologists study the history of the Earth in terms of periods characterized by different rock layers, cosmologists study the history of the Universe in epochs characterized by different matter content. Among these epochs shown in figure 2, the Universe has a beginning. This was a quick era during which the universe expanded such that its size increased acceleratingly and that this era gave rise to deviations in an overall smooth density distribution. These deviations, observed as temperature fluctuations in the sky by many telescopes today, later on led to formation of first galaxies. We call this initial stage the epoch of *inflation.* After inflation the rate of the expansion of the Universe was decelerated during radiation and matter domination. Interestingly today's Universe is once again undergoing accelerated expansion and this current epoch is referred to as the epoch of *dark energy domination.*

But how does one express all these different epochs? When we talk about different rates of expansion we are talking about different geometries for the spacetime. So the question is how does one express

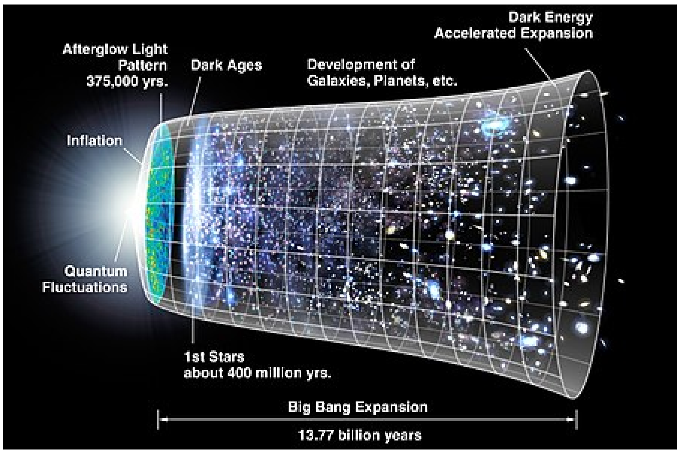


Figure 2: The cosmological epochs. Source: Wikipedia

spacetime? Remember the Pythagorean theorem from elementery school, for a triangle on a flat surface with two perpendicular sides of length A and B it gives the length C for the third side to be

$$C^2 = A^2 + B^2. \tag{1}$$

If you play with the angles of the triangle or if you draw it on a bent surface this formula will be adjusted to involve the angles. Actually the way of calculating distances is determined by the properties of the space in which the distance is being calculated. We categorize spacetimes by the expression for a small distance called the metric. For cosmic spacetimes this expression in with a time coordinate t and position coordinates $\vec{x}$ is written as

$$c^2 ds^2 = -c^2 dt^2 + a^2(t)\,(d\vec{x})^2\,. \tag{2}$$

Equation (2) is known as the Friedmann–Lemaître–Robertson–Walker metric after the first scientist who obtained it as a solution in General Relativity. The $-dt$ here means a small interval in time where the minus in front is what tells us that this is a time coordinate and c stands for the speed of light. The $+\,(d\vec{x})^2$ means a small distance in space. In the way we have written the spatial surfaces of this spacetime are flat like the surface of table, but because of the $a(t)$ in front of them the overall size of these spatial surfaces keep changing with time. The $a(t)$ is called the *scale factor* because it rescales the distances. If it had been $a(\vec{x})$, the spatial lengths would be changing from place to place. There being no spatial dependence in $a(t)$ carries the notion that this universe has homogeneous spatial surfaces. The fact that each direction is multiplied by the same scale factor $a(t)$ is the notion of isotropy, all directions are treated the same. And the fact that the scaling of spatial surfaces depends only on time carries the notion of the Universe being dynamic. For each of the cosmic epoch $a(t)$ is a different function. Changes in the description of the coordiantes that on the whole leave the expression (2) unchanged are called spacetime symmetries. Different epochs posses different symmetries, each of which are helpful to understand the phenomena that take place.

We started with quite picturesque descriptions of the Universe in this writing and moved to mathematical expressions like (2) which most of the readers are seeing for the first time in their lives. Mathematics is a very powerful tool in contemporary science today. This approach has been an evolving practice in science since Galileo Galilei, who thought that the suitable language to understand the workings of nature is the language of Mathematics. It has proven to be so since then. Yet each time a new mathematical formulation is established, most wonder how and when will any of the complicated looking expressions will become practical? The truth is no one at the time can know. Let us conclude our narration on how scientific ideas about the Universe are being weaved and are evolving across the centuries with a beautiful example on the matter of practicality that goes back to the works of Galilei himself.

Galilei was simply just curious about how bodies moved under a gravitational field. To study this he rolled spheres down inclined planes, on flat table tops and swung them at the end of a rope as uncomplicated exercises to think through. He tried to capture these motions with pen on paper as relationships between what described the system under question (the mass of the object, the inclination angle of the plane, the length of the rope) and the surroundings (the earth's gravitational pull, the friction around). In early 1600's, interested in pendulums, that is a sphere hanging at the end of a rope, he realized that there is a relationship between the length of the rope and the period with which the sphere keeps coming back to the position it started from. To simplify the mathematics, which were trigonometric relationships, he assumed that the sphere was swung from a small angle. This study was purely out of curiosity, the freedom to make assumptions led to less complicated expressions. By 1650's looking at Galilei's works, Christiaan Huygens realized that a pendulum would be a great keeper of time. He made the necessary corrections for more realistic angles, that would make the pendulum a practical clock in everyday life, and together with the clockmaker Salomon Coster they brought pendulum clocks to life, improving the accuracy of time keeping at the time. Just like what Galilei worked on had its practical applications much later on, the benefit of the complicated equations we work with today will come into play in our everyday lives in due time.